TRINITY COLLEGE

Mathematics Methods Units 3,4
Test 5 2019

Calculator Assumed

CRV, Normal Distribution and Intro to Sampling

STUDENT'S NAME SOLUTIONS

DATE: Friday 16 August **TIME:** 50 minutes **MARKS:** 49

INSTRUCTIONS:
Standard Items: Pens, pencils, drawing templates, eraser,

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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1. (2 marks)

A researcher is investigating the attitude towards Mathematical studies of secondary students in the Perth metropolitan area. The researcher chooses to survey the first 100 students that arrive to school at Trinity College. Describe the bias in the researcher's sample.

- student opinions at this school not necessarily representative of student population
- Students who arrive to school first may be more studious & therefore their attitude towards mathematics might be higher

2. (2 marks)

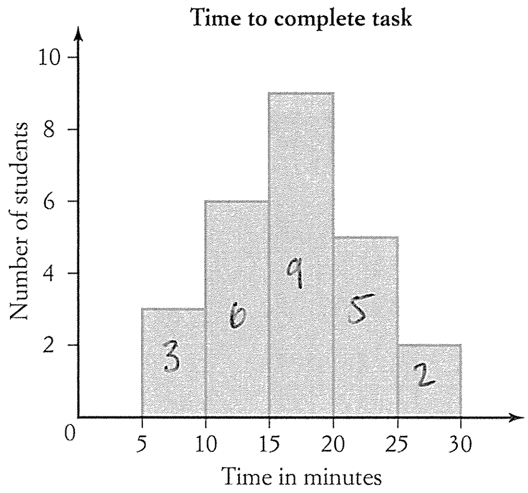
An opinion poll surveyed 1800 voters to determine the proportion of whom would vote for the Coalition over Labor on a two-party preferred basis.

Select the items from the list below that would be important features of the survey, for the results to be considered reflective of the national proportion.

- ☑ Ensure that the survey included a variety of age groups.
- ☐ Ensure that they asked people at the same time of day.
- ☐ Ensure that no two people with the same surname were selected.
- ☑ Ensure that the survey included voters from a variety of electorates.
- ☐ Ensure that no one related to employees of the opinion poll company were selected.
- ☐ Select the first 1800 names in a telephone directory.

3. (4 marks)

A group of year 12 Methods students were asked to completed an extended task on probability distributions. The time taken for each to complete the task is displayed by the histogram below.



Determine the probability that as student selected from this group completed the task in

(a) exactly 10 minutes. [1]

$= 0$

(b) less than 15 minutes. [1]

$= \frac{9}{25}$

(c) less than 25 minutes given they spent more than 10 minutes. [2]

$= \frac{20}{22}$

4. (8 marks)

The probability density function of a continuous random variable X is given by

$p(x) = \frac{24}{x^3}$ for $3 \leq x \leq 6$

(a) Determine the cumulative distribution function. [2]

$$= \begin{cases} 0 & x < 3 \\ -\frac{12}{x^2} & 3 \leq x \leq 6 \\ 1 & x > 6 \end{cases}$$

(b) Use the cumulative distribution function to determine $P(3 \leq X < 4)$. [2]

$$= \int_3^4 \frac{24}{x^3}$$

$$= \frac{7}{12}$$

(c) Determine the expected value of X. [1]

$$= \int_3^6 \frac{24}{x^3} \times x$$

$$= 4$$

(d) Determine the standard deviation of X. [1]

$$= \int_3^6 \frac{24}{x^3} \times x^2 \quad - 4^2 \qquad OR \qquad = \int_3^6 (x-4)^2 \times \frac{24}{x^3} dx$$

$$= \sqrt{0.6355} \qquad\qquad = \sqrt{0.6355}$$

$$= 0.7972 \qquad\qquad = 0.7972$$

(e) This continuous random variable X is transformed to the random variable Y according to the equation $Y = 4X - 2$.

Determine the:

(i) mean of Y. [1]

$$= 4 \times 4 - 2$$

$$= 14$$

(ii) standard deviation of Y. [1]

$$= 0.7972 \times 4$$

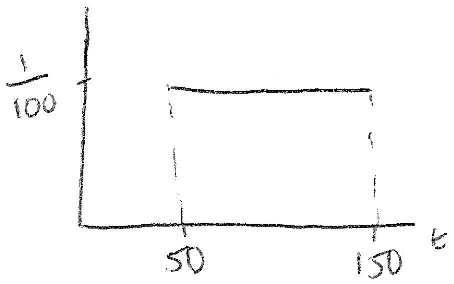
$$= 3.19$$

5. (8 marks)

The serving time, T seconds, for a customer at an automatic banking machine is a uniformly distributed random variable, with lower and upper limits 50 and 150. The serving times for different customers are independent.

(a) Sketch the graph of the distribution function for T and state the probability density function. [2]

$$P(T=t) = \begin{cases} \frac{1}{100} & 50 \leq t \leq 150 \\ 0 & \text{otherwise} \end{cases}$$



(b) Evaluate P(T ≥ 120). [1]

$$= \frac{30}{100}$$

(c) Determine the mean serving time. [1]

$$= 100$$

(d) Determine the variance in serving times. [2]

$$= \frac{(150-50)^2}{12}$$

$$= 833.\dot{3}$$

(e) Determine the value of k if $P(60 < X < k / X < k) = \frac{2}{3}$. [2]

$$\frac{P(60 < x < k)}{P(x < k)} = \frac{\frac{k-60}{100}}{\frac{k-50}{100}}$$

$$\frac{2}{3} = \frac{k-60}{k-50}$$

k = 80 seconds

6. (12 marks)

(a) Scientists have discovered that the leaves of a specific species of gum tree are normally distributed with a mean length of 14.7 cm and standard deviation of 3.6 cm.

(i) Determine the probability that a leaf selected from this type of gum tree has a length larger than 12.2 cm. [2]

$X \sim N(14.7, 3.6^2)$

$P(X > 12.2) = 0.7563$

(ii) Determine the probability that a leaf selected from this type of gum tree has a length between 12.2 and 14.7 cm if it is less than 15 cm. [2]

$= P(12.2 < X < 14.7 \mid X < 15) =$

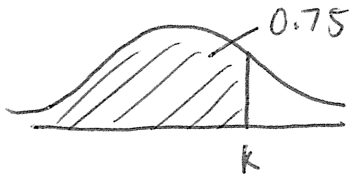
$= \dfrac{P(12.2 < X < 14.7)}{P(X < 15)}$

$= \dfrac{0.25629}{0.53321} = 0.4807$

(iii) Determine the 0.75 quantile length for leaves of this species of gum tree. [2]

$P(X < K) = 0.75$

$K = 17.13 cm$



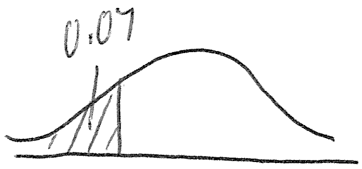
(iv) 10 leaves are randomly selected from this species of gum tree to be further analysed. Determine the probability that at least half of these leaves have a mean length between 12.2 and 14.7 cm. [3]

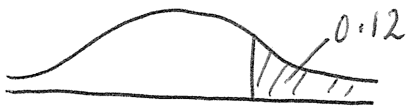
$Y \sim B(10, 0.25629)$

$P(Y \geq 5) = 0.0857$

(b) A different type of tree, the Eucalyptus, has 7% of its leaves less than 4 cm and 12% of its leaves greater than 14 cm.

Determine the mean and standard deviation of the Eucalyptus leaf length. (Assume the length of the Eucalyptus leaves are normally distributed). [3]





$P(W < 4) = 0.07$

$z_1 = \frac{4-\mu}{\sigma}$

$-1.476 = \frac{4-\mu}{\sigma}$

$P(W > 14) = 0.12$

$z_2 = \frac{14-\mu}{\sigma}$

$1.175 = \frac{14-\mu}{\sigma}$

$\mu = 9.57\,cm$

$\sigma = 3.77\,cm$

7. (9 marks)

The Applegg Company are selling 50 gram eggs to IGB, whose quality control officer has told the Applegg company that it has to satisfy various conditions in order to keep the contract.

The eggs are packed by a machine which chooses them from a conveyor belt. The machine weighs them to check their weight is 'close' to 50 grams. Past experience has shown that the weights of the eggs it chooses are normally distributed with a mean of 49.5 grams and a standard deviation of 1.26 grams.

(a) What proportion of eggs will be less than 48 grams? [1]

$X \sim N(49.5, 1.26^2)$

$P(X < 48) = 0.1169$

Egg-cartons contains 12 eggs and are packed into boxes containing 20 cartons each.

(b) An egg-carton is rated as faulty if it contains more than one egg that weighs less than 48 grams. What percentage of egg-cartons produced by this machine would be rated as faulty? [3]

$Y \sim B(12, 0.1169)$

$P(Y > 1) = 0.4177$

(c) The egg-cartons are now packed into boxes. What is the probability that the box contains no faulty cartons? [2]

$T \sim B(20, 0.4177)$

$P(T = 0) = 0.00002$

(d) IGB quality control recognizes that a perfect record is impossible so they set a standard that at least 90% of boxes contain no faulty egg-cartons. What is the largest allowable probability of an egg-carton being faulty? [3]

$U \sim B(20, p)$

$P(T = 0) \geq 0.90$

$p = 0.0053$

8. (4 marks)

A continuous random variable, T, has a distribution modelled by:

$$f(t) = \begin{Bmatrix} ke^{-kt} & ; t \geq 0 \\ 0 & ; otherwise \end{Bmatrix}$$

Determine k (to 4 decimal places) if the probability that T is greater than 10 given that it is greater than 4 is 0.5

$$P(T > 10 \mid T > 4) = 0.5$$

$$\frac{P(T > 10)}{P(T > 4)} = 0.5$$

$$\frac{\int_{10}^{\infty} ke^{-kt}\,dt}{\int_{4}^{\infty} ke^{-kt}\,dt} = 0.5$$

$$\frac{\left[-e^{-kt}\right]_{10}^{\infty}}{\left[-e^{-kt}\right]_{4}^{\infty}} = 0.5$$

$$\frac{-e^{-\infty} - (-e^{-10k})}{-e^{-\infty} - (-e^{-4k})} = 0.5$$

$$K = 0.1155$$